Review 2

6) $f(x) = 2x^3 - 15x^2 + 36x - 32$

$f'(x) = 6x^2 - 30x + 36$ $\quad 6(x^2 - 5x + 6) = 6(x-2)(x-3) = 0$

$f''(x) = 12x - 30$ $\quad x = 2 \quad x = 3$

$y = 2(2)^3 - 15(2)^2 + 36(2) - 32$

$16 - 60 + 72 - 32 = -4 \quad (2, -4)$

$y = 2(3)^3 - 15(3)^2 + 36(3) - 32$

$54 - 135 + 108 - 32 = -5 \quad (3, -5)$

$y'' = 12(2) - 30 = -6 < 0$ Max @ $(2, -4)$

$y'' = 12(3) - 30 = 6 > 0$ Min @ $(3, -5)$

P.O.I.) $0 = 12x - 30 \quad \frac{30}{12} = x = \frac{5}{2}$

y-coordinate) $y = 2\left(\frac{5}{2}\right)^3 - 15\left(\frac{5}{2}\right)^2 + 36\left(\frac{5}{2}\right) - 32$

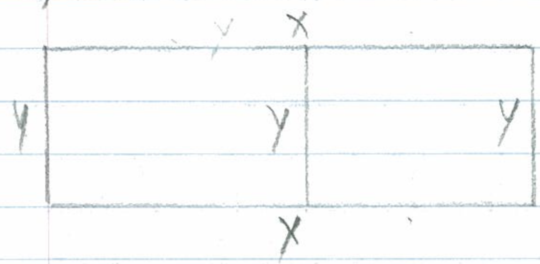
$y = 2\left(\frac{125}{8}\right) - 15\left(\frac{25}{4}\right) + 90 - 32$

$y = \frac{125}{4} - \frac{375}{4} + 90 - 32 = -\frac{250}{4} + 58$

$y = \frac{-18}{4} = -\frac{9}{2} \quad \left(\frac{5}{2}, \frac{-9}{2}\right)$

Hilroy

age 734 10) 6000m



$3y + 2x = 6000m$

$xy = A$

$y = \dfrac{6000 - 2x}{3}$

$$x\left(\frac{6000-2x}{3}\right) = A$$

$$\frac{6000x - 2x^2}{3} = A$$

$$A = 2000x - \tfrac{2}{3}x^2$$

$A' = 2000 - \tfrac{4}{3}x$

$0 = 2000 - \tfrac{4}{3}x$

$\tfrac{4}{3}x = 2000$

$x = \dfrac{3(2000)}{4}$ $\qquad x = 1500$

$3y + 2(1500) = 6000m$

$3y = 3000m$

$y = 1000m$

$1000m \cdot 1500m = 1500\,000m^2$

Chapter 23 practice Test page 693

2) $\lim_{x \to \infty} \frac{1-4x^2}{x+2x^2} = \frac{(1-2x)\cancel{(1+2x)}}{x\cancel{(1+2x)}} = \frac{1-2x}{x}$

$= -2$

3) $y = 3x^2 - \frac{4}{x^2}$ $(2, 11)$

$y = 3x^2 - 4x^{-2} = y' = 6x + 8x^{-3} = y' = 6x + \frac{8}{x^3}$

$y' = 6(2) + \frac{8}{2^3}$ $y' = 13$ y' = slope of tangent line solve for y'

5) $y = 4x^6 - 2x^4 + \pi^3$ $y' = 24x^5 - 8x^3$

6) $y = 2x(5-3x)^4$

$u = 5-3x$

$u' = -3$

$f(x) = 2x$

$f'(x) = 2$

$g(x) = u^4$

$g'(x) = -3(4)u^3$

$g'(x) = -12u^3$

$y = (2x)(u^4)$

$y' = 2(u^4) + 2x(-12)(u^3)$

$y' = 2(5-3x)^4 + -24x(5-3x)^3$

7) $(1+y^2)^3 - x^2 y = 7x$

$u = 1+y^2$

$u' = 2y\frac{dy}{dx}$

$f(x) = x^2$

$f'(x) = 2x$

$g(x) = y$

$g'(x) = \frac{dy}{dx}$

$2y\frac{dy}{dx}(3)(u)^2 - \left[(x^2)\frac{dy}{dx} + y(2x)\right] = 7$

$6y\frac{dy}{dx}(1+y^2)^2 - x^2\frac{dy}{dx} - 2xy = 7$

$6y\frac{dy}{dx}(u)^2 - x^2\frac{dy}{dx} = 7 + 2xy$ $\left(6y(u^2) - x^2\right)\frac{dy}{dx} = 7 + 2xy$

$\frac{dy}{dx} = \frac{7+2xy}{6y(1+y)^2 - x^2}$

Product Rule

$F' = f(x)\,g'(x) + g(x)\,f'(x)$

Quotient Rule

$\dfrac{f(x)}{g(x)} \qquad F' = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Power Rule

$f(x) = 10(1-4x^2)^5$

$f'(x) = 50(1-4x^2)^4(-8x)$

$= -400x(1-4x^2)^4$

page 691: 35) $(2x-3y)^3 = x^2 - y$

$u = 2x - 3y$

$u' = 2 - 3\dfrac{dy}{dx}$

$\left(2 - 3\dfrac{dy}{dx}\right)(3)(2x-3y)^2 = 2x - \dfrac{dy}{dx}$

$6 - 9\dfrac{dy}{dx}(2x-3y)^2 = 2x - \dfrac{dy}{dx}$

$6(2x-3y)^2 - 2x = 9\dfrac{dy}{dx}(2x-3y)^2 - \dfrac{dy}{dx}$

$\left(9(2x-3y)^2 - 1\right)\dfrac{dy}{dx}$

$-\dfrac{dy}{dx} = \dfrac{6(2x-3y)^2 - 2x}{9(2x-3y)^2 - 1}$

$\dfrac{dy}{dx} = \dfrac{2x - 6(2x-3y)^2}{1 - 9(2x-3y)^2}$

#9, 36

693 641

Chapter 24

Find y' and y''

i) set $y'=0$ to find min/max

iii) $y''<0 \rightarrow$ Max

$y''>0 \rightarrow$ min

$y''=0 \rightarrow$ Failed (can't use this check)

iv) P.O.I set $y''=0$ solve for x and evaluate in original equation

page 734 #7

7) $y = x^3 + 6x^2$

$y' = 3x^2 + 12x$

$y'' = 6x + 12$

i) $y'=0$

$0 = 3x^2 + 12x$

$0 = 3x(x+4)$

$x=0 \quad x=-4$

ii) $y = x^3 + 6x^2$

$y = 0^3 + 6(0)^2 \quad y=0 \quad (0,0)$

$y = -4^3 + 6(-4)^2 \quad y = 32 \quad (-4, 32)$

iii) $y'' = 6(0) + 12 = 12 > 0 \therefore$ min at 0

$y'' = 6(-4) + 12 = -12 < 0 \therefore$ max at -4

iv) P.O.I.

$0 = 6x + 12$

$-12 = 6x$

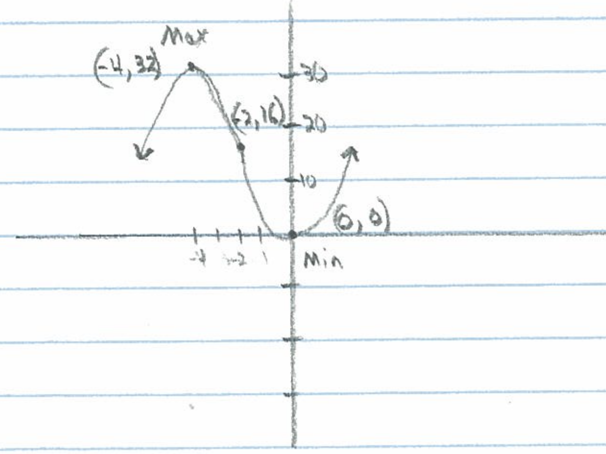
$x = -2$

$y = -2^3 + 6(-2)^2$

$y = -8 + 24$

$y = 16$

$(-2, 16)$



Hilroy

Review 1

1) $\lim\limits_{x \to -4} \dfrac{x^2-16}{x+4} = \dfrac{(x-4)\cancel{(x+4)}}{\cancel{(x+4)}} = x-4 = -4-4 = -8$

2) $\lim\limits_{x \to 0} \dfrac{3x^3-2x^2-5x}{x^2+x} = \dfrac{x(3x^2-2x-5)}{x(x+1)} = \dfrac{\cancel{x}(3x-5)\cancel{(x+1)}}{\cancel{x}\cancel{(x+1)}}$

$= 3x-5 = 3(0)-5 = -5$

3) $\lim\limits_{x \to -3} \dfrac{x^3+27}{x+3} \quad \dfrac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}} = x^2-3x+9$

$= (-3)^2 - 3(-3) + 9 = 9+9+9 = 27$

4) $\lim\limits_{x \to \infty} \dfrac{x^2-3x+2}{3x^2-6} \cdot \dfrac{1/x^2}{1/x^2} = \dfrac{\cancel{x^2/x^2}^{\,1} - \cancel{3x/x^2}^{\,0} + \cancel{2/x^2}^{\,0}}{\cancel{3x^2/x^2}^{\,3} - \cancel{6/x^2}^{\,0}} = \dfrac{1}{3}$

5) $\lim\limits_{x \to \infty} \dfrac{x^2-7x+13}{x^3-6x^2+6} \cdot \dfrac{1/x^3}{1/x^3} = \dfrac{\cancel{x^2/x^3}^{\,0} - \cancel{7x/x^3}^{\,0} + \cancel{13/x^3}^{\,0}}{\cancel{x^3/x^3}^{\,1} - \cancel{6x^2/x^3}^{\,0} + \cancel{6/x^3}^{\,0}} = \dfrac{0}{1} = 0$

6) $\lim\limits_{x \to 2} \dfrac{x^4-16}{x-2} \quad \dfrac{\cancel{(x-2)}(x+2)(x^2+4)}{\cancel{(x-2)}} = (2+2)(2^2+4)$

$4 \cdot 8 = 32$

7) $f(x) = 3x^2 - 2x^{-1} + x^{3/2}$

$f'(x) = 6x + 2x^{-2} + \tfrac{3}{2}x^{1/2} = 6x + \dfrac{2}{x^2} + \dfrac{3\sqrt{x}}{2}$

8) $y = (3x-3)(\sqrt{x}+5) \qquad y = 3x^{3/2} + 15x - 3x^{1/2} - 15$

$y' = (\tfrac{3}{2} \cdot 3)x^{1/2} + 15 - \tfrac{3}{2}x^{-1/2} = \dfrac{9\sqrt{x}}{2} - \dfrac{3}{2\sqrt{x}} + 15$

8) $y = (3x-3)(x^{1/2}+5)$

$f(x) = 3x-3$

$g(x) = x^{1/2}+5$

$g'(x) = \frac{1}{2}x^{-1/2}$

$f'(x) = 3$

$y' = 3(x^{1/2}+5) + (3x-3)\left(\frac{1}{2\sqrt{x}}\right)$

9) $f(x) = \frac{2x^3-x+4}{5x-6}$

$f(x) = 2x^3-x+4$

$g(x) = 5x-6$

$g'(x) = 5$

$f'(x) = 6x^2-1$

$= \frac{(5x-6)(6x^2-1)-(2x^3-x+4)(5)}{(5x-6)^2}$

$= \cancel{30}x^3 \cancel{-5x} - 36x^2 + 6 - \cancel{10x^3} \cancel{+5x} - 20$

$\frac{20x^3-36x^2-14}{(5x-6)^2}$

10) $f(x) = \frac{1}{(5-2x)^4}$

$f(x) = 1$

$f'(x) = 0$

$g(x) = (5-2x)^4$

$g'(x) = -8(5-2x)^3$

$\frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$= \frac{0(5-2x)^4 - (1)-8(5-2x)^3}{(5-2x)^8} = \frac{8\cancel{(5-2x)^3}}{(5-2x)^{\cancel{8}}} = \frac{8}{(5-2x)^5}$

11) $y = \sqrt{2x^3 - 5x^2 + x}$

$u = 2x^3 - 5x^2 + x$

$du = 6x^2 - 10x + 1$

$y = (2x^3 - 5x^2 + x)^{1/2}$

$y' = (u)^{-1/2} \, \frac{1}{2} \, du$

$y' = \dfrac{6x^2 - 10x + 1}{2\sqrt{2x^3 - 5x^2 + x}}$

12) $f(x) = (3x+2)^5 (2x^2 - x + 5)$

$f(x) = (3x+2)^5$

$f'(x) = 15(3x+2)^4$

$g(x) = 2x^2 - x + 5$

$g'(x) = 4x - 1$

$f'(x) = (2x^2 - x + 5) 15 (3x+2)^4 + (3x+2)^5 (4x-1)$

13) $2x^2 + 3y^2 + x - 4y - 1 = 0$

$4x + 6y\dfrac{dy}{dx} + 1 - 4\dfrac{dy}{dx} = 0$

$(6y - 4)\dfrac{dy}{dx} = -4x - 1$

$\dfrac{dy}{dx} = \dfrac{-4x - 1}{6y - 4}$

14) $2xy - y^2 = 1$

$f(x) = 2x$

$f'(x) = 2$

$g(x) = y$

$g'(x) = \dfrac{dy}{dx}$

$2y + 2x\dfrac{dy}{dx} - 2y\dfrac{dy}{dx} = 0$

$(2x - 2y)\dfrac{dy}{dx} = -2y$

$\dfrac{dy}{dx} = \dfrac{-2y}{2x - 2y}$ $\quad \dfrac{dy}{dx} = \dfrac{-y}{x - y}$

15) $3x^3 - xy^2 - 2y = 0$

$f(x) = -x$

$f'(x) = -1$

$g(x) = y^2$

$g'(x) = 2y\frac{dy}{dx}$

$9x^2 + y^2(-1) + (-x)\,2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$9x^2 - y^2 - 2xy\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$(-2xy - 2)\frac{dy}{dx} = -9x^2 + y^2$

$\frac{dy}{dx} = \frac{-9x^2 + y^2}{-2xy - 2}$

16) $x^2y + 5x^2 = 3y^2 + 4$

$f(x) = x^2$

$f'(x) = 2x$

$g(x) = y$

$g'(x) = \frac{dy}{dx}$

$(y)(2x) + x^2\frac{dy}{dx} + 10x = 6y\frac{dy}{dx}$

$2xy + 10x = (6y - x^2)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2xy + 10x}{6y - x^2}$

Review 2

1) $y = 9x^{5/2} - 7x^{3/2}$ $\quad x = 4$

$y' = 5/2(9)x^{5/2 - 2/2} - 7(3/2)x^{3/2 - 2/2}$

$y' = 45/2 x^{3/2} - 21/2 x^{1/2}$ $\quad y' = \frac{45}{2}(4)^{3/2} - \frac{21}{2}4^{1/2}$

$y' = 180 - 21 = 159$

2) $f(x) = x\sqrt{x^2+1} = x(x^2+1)^{1/2}$

$u = x^2+1$

$du = 2x$

$1(x^2+1)^{1/2} + x(x)(x^2+1)^{-1/2}$

$(x^2+1)^{1/2} + \frac{x^2}{(x^2+1)^{1/2}}$

$f(x) = x$

$f'(x) = 1$

$g(x) = (x^2+1)^{1/2}$

$g'(x) = 2x(1/2)(x^2+1)^{-1/2}$

3) $f(x) = \frac{1}{2}x^6 - \frac{1}{6}x^3 + 3x$

$f'(x) = 3x^5 - \frac{1}{2}x^2 + 3$

$f''(x) = 15x^4 - x$

$f'''(x) = 60x^3 - 1$

4) $y = 7(x)^{1/2} - 3x^{-1}$

$\frac{dy}{dx} = \frac{\frac{7}{2}}{x^{1/2}} + \frac{3}{x^2}$ $\quad dy = \left[\frac{7}{2\sqrt{x}} + \frac{3}{x^2}\right]dx$

Hilroy

5) $y = \sqrt{x} + 2$ $\qquad x = 9$

$y' = \frac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2(x)^{\frac{1}{2}}} = \dfrac{1}{6}$

$y = \sqrt{9} + 2 \qquad (9, 5)$

$y - 5 = \frac{1}{6}(x - 9) \qquad y = \frac{1}{6}x - \frac{9}{6} + 5$

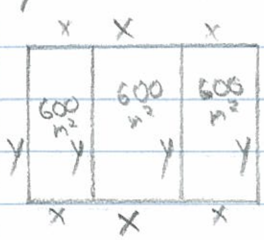
$y = \frac{1}{6}x + \frac{21}{6}$ Tangent Line

Perpendicular slope

$m_n = -\dfrac{1}{m} = -\dfrac{1}{\frac{1}{6}} = -6$

Normal Line

$y - 5 = -6(x - 9)$
$y = -6x + 54 + 5$
$y = -6x + 59$

7) 

$P = 4y + 6x$
$A = 600m^2$
$600m^2 = xy \qquad y = \dfrac{600}{x}$

$P = 4\left(\dfrac{600}{x}\right) + 6x$

$P = \dfrac{2400}{x} + 6x$

$P = 2400x^{-1} + 6x \qquad -6x^2 = -2400 \qquad y = \dfrac{600}{x}$
$P' = -2400x^{-2} + 6 \qquad x^2 = 400$
$0 = \dfrac{-2400}{x^2} + 6 \qquad x = 20 \qquad y = 30$

$p = 4(30) + 6(20) = 240\,m$

8) $V = 576 cm^3$

$L = 2w$

$576 cm^3 = L \cdot w \cdot h$

$576 cm^3 = 2w \cdot w \cdot h$

$576 cm^3 = 2w^2 \cdot h$

$\frac{576}{2w^2} = h \qquad h = \frac{288}{w^2}$

$SA = 2(2wh) + 2wh + (2w^2)2$

$SA = 6wh + 4w^2$

$SA = 6\frac{288}{w^2}w + 4w^2$

$SA = \frac{1728}{w} + 4w^2 \qquad SA' = -1728w^{-2} + 8w$

$0 = -1728w^{-2} + 8w$

$-8w = -1728w^{-2}$

$-8w = \frac{-1728}{w^2}$

$-8w^3 = -1728$

$w^3 = \frac{-1728}{-8} \qquad w^3 = 216$

$w = 6$

$L = 2w$

$L = 2(6)$

$L = 12$

$h = \frac{288}{36} \qquad h = 8$

page
159

2) $\int x\sqrt{1-2x^2}\,dx$ $\qquad u = 1-2x^2$

$du = -4x\,dx$

$\int x(1-2x^2)^{1/2}\,dx$

$\frac{1}{-4}\int x\,(u)^{1/2}\,dx\,(-4)$

$\frac{1}{-4}\int u^{1/2}\,du \quad = \frac{1}{-4}\left[\frac{u^{1/2+1}}{3/2}\right] \quad = \frac{1}{-4}\,\frac{2(1-2x^2)^{3/2}}{3}$

$= -\frac{(1-2x^2)^{3/2}}{6} + C$

3) $\frac{dy}{dx} = (6-x)^4$ $\qquad$ Find y in terms of x at a point (5,2)

$dx(-)\int(6-x)^4\,dx$

$u = 6-x$

$du = -1$

$-\int u^4\,du$

$-\frac{u^{4+1}}{5} \quad = \quad -\frac{(6-x)^5}{5} + C$

Solve for C

$2 = -\frac{1}{5}(6-5)^5 + C \qquad 2 = -\frac{1}{5}(1) + C$

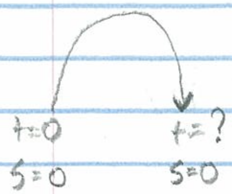
$-C = -\frac{1}{5} - \frac{2\cdot 5}{5}$

$-C = -\frac{11}{5} \qquad C = \frac{11}{5}$

$y = -\frac{1}{5}(6-x)^5 + \frac{11}{5}$

9) $s(t) = 6.25 + 9.73(t) - 4.90(t)^2$

a) Time ball hits ground



set $s(t)=0$

$0 = -4.90t^2 + 9.73t + 6.25$

$= \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{-9.73 \pm \sqrt{9.73^2-(-4.90)6.25(4)}}{-9.8}$

$= \dfrac{-9.73 \pm 14.74}{-9.8} = 2.5$ sec

b) Velocity at impact

$s'(t) = v(t)$

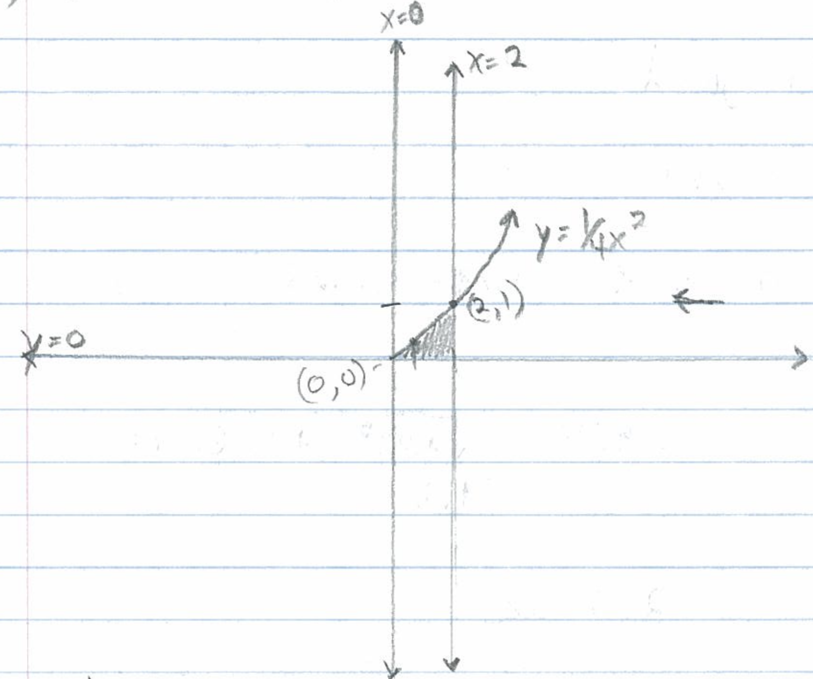
$s'(t) = 9.73 - 9.80t$

$v(t) = 9.73 - 9.80(2.5)$

$v(t) = -14.8$ m/s

page 796

1) $y = \frac{1}{4}x^2 \quad y = 0 \ , \ x = 2$



$x = \sqrt{4y}$

Find area

$A = \int_a^b (y_2 - y_1)\,dx \qquad y_2 = \frac{1}{4}x^2 \quad y_1 = 0$

$A = \int_0^2 (\frac{1}{4}x^2 - 0)\,dx \qquad A = \int_0^2 \frac{1}{4}x^2\,dx$

$A = \frac{\frac{1}{4}x^{2+1}}{3} \qquad A = \frac{x^3}{12}\Big|_0^2 \qquad A = \frac{2^3}{12} - \frac{0^3}{12}$

$A = \frac{8}{12} = \frac{2}{3}$ units^2

2) $\bar{x} = \dfrac{\int_a^b x(y_2 - y_1)\,dx}{\frac{2}{3}} \qquad \bar{y} = \dfrac{\int_a^b y(x_2 - x_1)\,dy}{\frac{2}{3}}$

$\bar{x} = \int_0^2 x\left(\frac{1}{4}x^2 - 0\right)dx$

$\bar{x} = \int_0^2 \frac{1}{4}x^3 \, dx$

$\bar{x} = \frac{x^{3+1}}{4(4)} = \frac{x^4}{16}\Big|_0^2 \qquad \bar{x} = \frac{16}{16} - \frac{0^4}{16} = 1$

$\bar{x} = \frac{1}{\frac{2}{3}} \qquad \bar{x} = \frac{1}{1} \cdot \frac{3}{2} \qquad \bar{x} = 3/2$

$\bar{y} = \int_0^1 y\left(2 - \sqrt{4y}\right)dy \qquad \bar{y} = \int_0^1 2y - 2y^{3/2}\,dy$

$\bar{y} = 2\int_0^1 y - y^{3/2}\,dy \qquad \bar{y} = 2\left(\frac{y^2}{2} - \frac{y^{5/2}}{5/2}\right)$

$\bar{y} = 2\left(\frac{y^2}{2} - \frac{2y^{5/2}}{5}\right)\Big|_0^1 \qquad \bar{y} = 2\left(\frac{1^2}{2} - \frac{2(1)^{5/2}}{5}\right)$

$\bar{y} = 2\left(\frac{1}{2} - \frac{2}{5}\right) = 2\left(\frac{5}{10} - \frac{4}{10}\right) = \frac{\frac{1}{5}}{\frac{2}{3}} = \frac{1}{5} \cdot \frac{3}{2} = \frac{3}{10}$

Centroid $= \left(\frac{3}{2}, \frac{3}{10}\right)$

Review 3

1) a) $\int \left(5 - \frac{6}{x^2} + 3x^5\right) dx$

a) $\frac{5x^{0+1}}{1} - \frac{6x^{-2+1}}{-1} + \frac{3x^{5+1}}{6} = 5x + \frac{6}{x} + \frac{x^6}{2} + C$

b) $\int (x\sqrt{x^3} + 3) \cdot dx = \int \left[(x) x^{3/2} + 3\right]$

$\int \left[x^{2/2} \cdot x^{3/2} + 3\right] dx = \int (x^{5/2} + 3) dx$

$= \frac{x^{5/2 + 2/2}}{7/2} + 3x = \frac{2\sqrt{x^7}}{7} + 3x + C$

c) $\int \left[\frac{x}{(3-7x^2)^5}\right] dx = \int \left[x(3-7x^2)^{-5}\right] dx$

$u = 3 - 7x^2$
$du = -14x\,dx$

$= \int \left[x (u)^{-5}\right] dx$

$= -\frac{1}{14} \int u^{-5} du$

$= \frac{-1}{14} \left(\frac{u^{-5+1}}{-4}\right) = \frac{-1}{14} \frac{(3-7x^2)^{-4}}{-4}$

$= \frac{1}{56(3-7x^2)^4} + C$

d) $\int \left[9z(3z^2-7)^{1/2}\right] dz$ $\qquad u = 3z^2 - 7$

$du = 6z\,dz$

$\int 9z(u^{1/2})\,dz = \frac{9}{6}\int u^{1/2}\,du$

$= \frac{3}{2}\left(\frac{u^{1/2+2/2}}{3/2}\right) = \frac{\cancel{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}} u^{3/2} = (3z^2-7)^{3/2} + C$

2) $f(x)$ if $\frac{dy}{dx} = 6x^2 - 3$ $\qquad (1,4)$

$f(x) = \int 6x^2 - 3\,dx = \frac{6x^{2+1}}{3} - \frac{3x^{0+1}}{1} = 2x^3 - 3x + C$

$4 = 2(1)^3 - 3(1) + C$ $\qquad -C = 2 - 3 - 4$ $\qquad C = 5$

$f(x) = 2x^3 - 3x + 5$

3) a) $\int_{-1}^{2} (4x - 6x^2)\,dx = \frac{4x^{1+1}}{2} - \frac{6x^{2+1}}{3}\Big|_{-1}^{2}$

$= 2x^2 - 2x^3 \Big|_{-1}^{2} = \left[2(2)^2 - 2(2)^3\right] - \left[2(-1)^2 - 2(-1)^3\right]$

$= 8 - 16 - 2 - 2 = -12$

b) $\int_1^2 \left(4x^3 - \frac{8}{x^5}\right) dx = \frac{4x^{3+1}}{4} - \frac{8x^{-5+1}}{-4}$

$= x^4 + \frac{2}{x^4} \Big|_1^2 = \left[(2)^4 + \frac{2}{(2)^4}\right] - \left[(1)^4 + \frac{2}{(1)^4}\right]$

$= 16 + \frac{1}{8} - 1 - 2 = 13 + \frac{1}{8} = \frac{105}{8}$

c) $\int_1^2 8x(x^2-1)^3 dx$ $\qquad u = x^2 - 1$

$\qquad du = 2x\,dx$

$\frac{8}{2}\int_1^2 u^3 du = 4\left(\frac{u^{3+1}}{4}\right) = (x^2-1)^4 \Big|_1^2$

$(2^2-1)^4 - \left[\cancel{(1^2-1)^4}^{\,0}\right] = 81$

d) $\int_0^4 2x\sqrt{16-x^2}\,dx = \int_0^4 2x(16-x^2)^{1/2} dx$ $\qquad u = 16 - x^2$

$\qquad du = -2x\,dx$

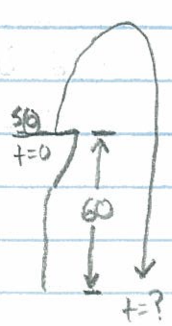
$\frac{2}{-2}\int_0^4 u^{1/2} du = -1\left(\frac{u^{1/2+2/2}}{3/2}\right) = \frac{-2(16-x^2)^{3/2}}{3}\Big|_0^4$

$= -2\left(\cancel{\frac{16-4^2}{3}}\right)^{3/2} - \left[\frac{-2(16-0^2)^{3/2}}{3}\right] = \frac{128}{3}$

4) $V_0 = 18.2\,m/s$ $\qquad A = -9.80\,m/s^2$ $\qquad V(t) = s'(t)$

$A(t) = v'(t)$

a)



$v(t) = \int -9.80\,dt$

$v(t) = \frac{-9.80t^{0+1}}{1} + C$

$v(t) = -9.80t + C$

$V(0) = 18.2 = -9.80(0) + C$

$C = 18.2$

$V(t) = -9.80t + 18.2$

$s(t) = \int (-9.80t + 18.2)\,dt = \frac{-9.80t^{1+1}}{2} + \frac{18.2t^{0+1}}{1} + C$

$s(t) = -4.90t^2 + 18.2t + C$

$s(0) = 60 = -4.90(0)^2 + 18.2(0) + C$

$C = 60$

$s(t) = -4.90t^2 + 18.2t + 60$ set $s(t) = 0$ and solve using quadratic equation

$0 = -4.90t^2 + 18.2t + 60$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-18.2 \pm \sqrt{18.2^2 - (-4.90)(4)(60)}}{-9.80}$

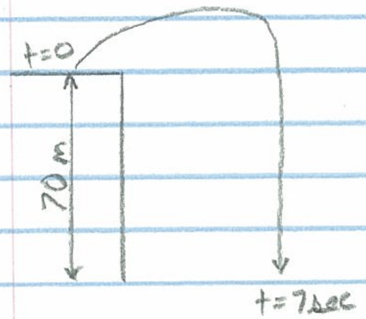
$= \frac{-18.2 \pm 38.8}{-9.80}$ $t = 5.82\,sec$

b) V_{Impact}

$V(t) = -9.80t + 18.2$ $V(5.82) = -9.80(5.82) + 18.2$

$V_{Impact} = -38.82\ m/s$

5) $A = -9.80\,m/s^2$



$s(t) = 0$ @ $t = 7$

$s(0) = 70$

$V(t) = \int -9.80\,dt$

$V(t) = \frac{-9.80t^{0+1}}{1} + C$

$V(t) = -9.80t + C_1$

$s(t) = \int -9.80t + C_1$

$s(t) = \frac{-9.80t^{1+1}}{2} + C_1 t^{0+1}$

$s(t) = -4.90t^2 + C_1 t + C_2$

$s(0) = 70 = \cancel{-4.90(0)^2}^{\,0} + \cancel{C_1(0)}^{\,0} + C_2$

$C_2 = 70$

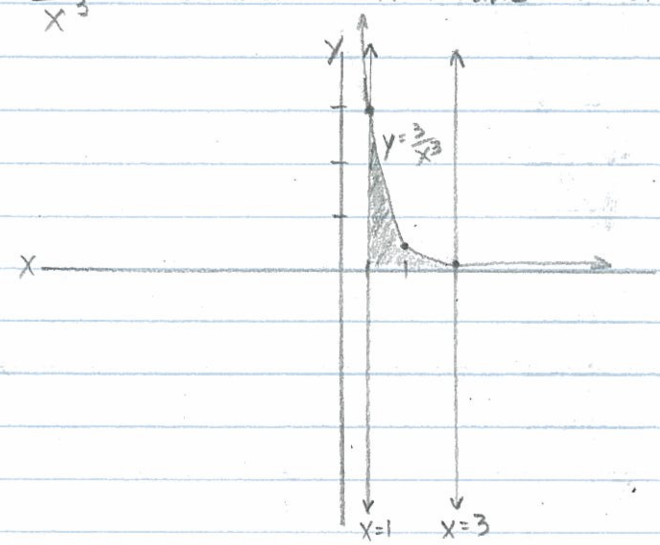
$s(t) = -4.90t^2 + C_1 t + 70$

$0 = -4.90(7)^2 + C_1(7) + 70 \rightarrow \frac{2401-70}{7} = C_1 = 24.3\,m/s$

$V_{Initial} = 24.3\,m/s$

Review 4

1) $y = \frac{3}{x^3}$ between $x = 1$ and $x = 3$

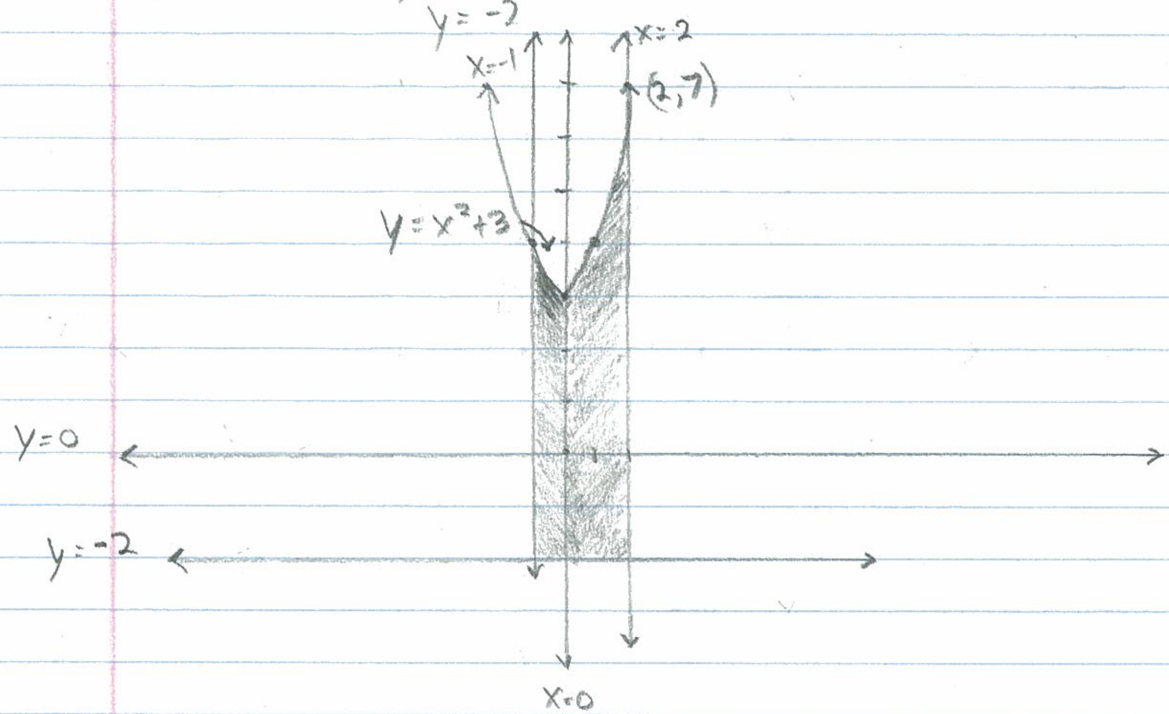


$A = \int_a^b (y_2 - y_1)\,dx \quad \cdot \quad A = \int_1^3 (3x^{-3} - 0)\,dx$

$A = \frac{3x^{-3+1}}{-2} = -\frac{3}{2x^2}\Big|_1^3 = -\frac{3}{2(3)^2} - \left[\frac{-3}{2(1)^2}\right] = \frac{-3}{18} + \frac{3}{2}$

$A = \frac{4}{3}$ units^2

2) $y = x^2 + 3 \qquad y + 2 = 0 \qquad x = -1 \qquad x = 2$



$A = \int_a^b (y_2 - y_1)\,dx \qquad A = \int_{-1}^{2} \left(x^2 + 3 - (-2)\right) dx$

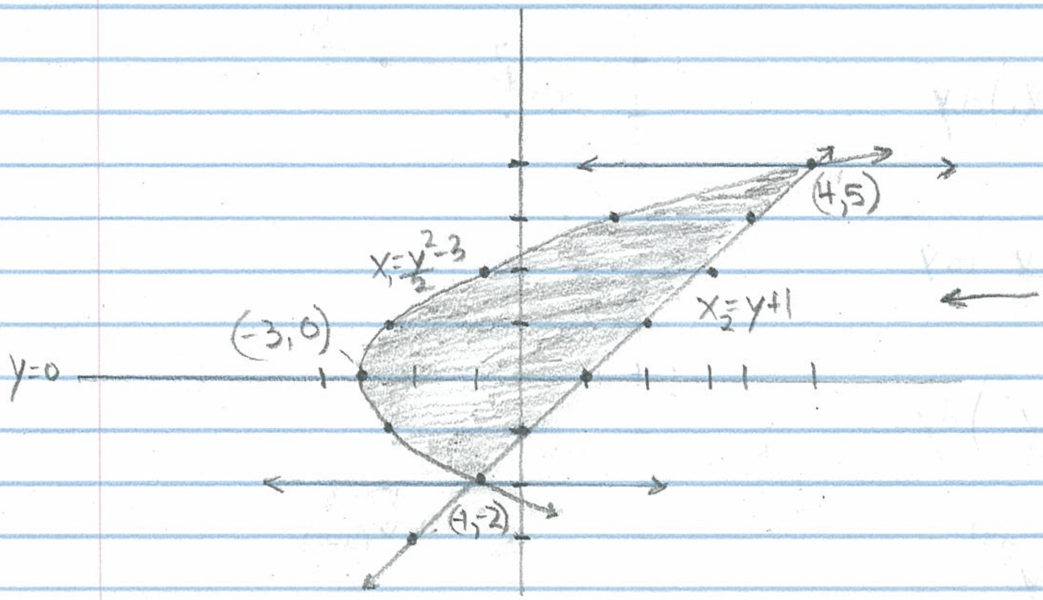
$A = \int_{-1}^{2} (x^2 + 5) \qquad A = \frac{x^{2+1}}{3} + \frac{5x^{0+1}}{1} = \frac{x^3}{3} + 5x \Big|_{-1}^{2}$

$\frac{2^3}{3} + 5(2) - \left[\frac{(-1)^3}{3} + 5(-1)\right] = \frac{8}{3} + 10 + \frac{1}{3} + 5$

$\frac{9}{3} + 15 = 18 \text{ units}^2$

3) $y^2 = 2x + 6$ and $y = x - 1$

$x = \frac{y^2}{2} - 3$ and $x = y + 1$



$$A = \int_a^b (x_2 - x_1)\, dy \qquad A = \int_{-2}^{4} \left[(y+1) - \left(\frac{y^2}{2} - 3\right)\right] dy$$

$$A = \int_{-2}^{4} \left(y + 1 - \frac{y^2}{2} + 3\right) dy \qquad A = \int_{-2}^{4} \left(\frac{-y^2}{2} + y + 4\right) dy$$

$$A = -\frac{y^{2+1}}{2(3)} + \frac{y^{1+1}}{2} + \frac{4y^{0+1}}{1} = \frac{-y^3}{6} + \frac{y^2}{2} + 4y \Big|_{-2}^{4}$$

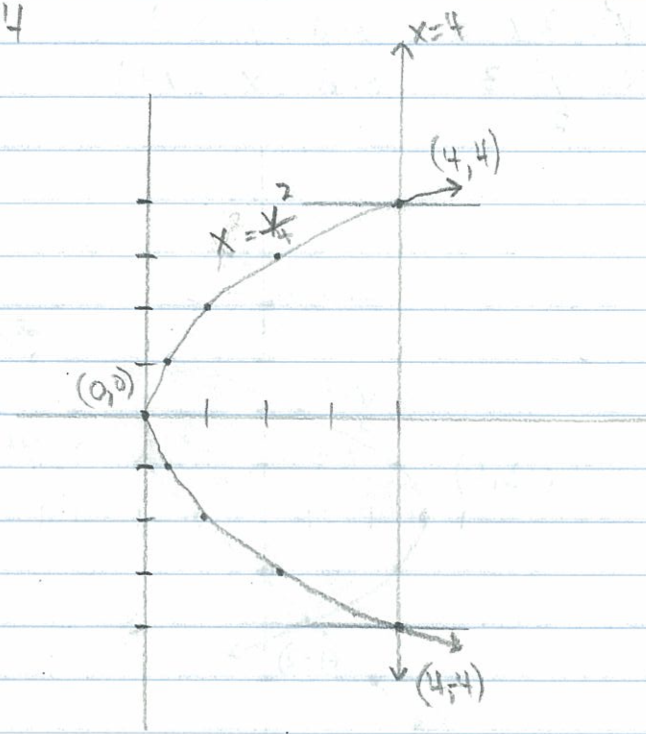
$$A = -\frac{(4)^3}{6} + \frac{(4)^2}{2} + 4(4) - \left[\frac{-(-2)^3}{6} + \frac{(-2)^2}{2} + 4(-2)\right]$$

$$A = -\frac{64}{6} + \frac{16}{2} + 16 - \frac{8}{6} - 2 + 8$$

$$A = -\frac{32}{3} - \frac{4}{3} + 30 \qquad A = 18 \text{ units}^2$$

4) $y^2 = 4x$ and $x = 4$

$x = \frac{y^2}{4}$



$A = \int_a^b (x_2 - x_1)\,dy$

$A = \int_a^b \left(4 - \frac{y^2}{4}\right) dy$

$A = \int_{-4}^{4} \left(4 - \frac{y^2}{4}\right) dy$

$A = \frac{4y^{0+1}}{1} - \frac{y^{2+1}}{4(3)}$

$A = 4y - \frac{y^3}{12} \Big|_{-4}^{4}$ $\quad A = 4(4) - \frac{(4)^3}{12} - \left[4(-4) - \frac{(-4)^3}{12}\right]$

$A = 16 - \frac{64}{12} + 16 - \frac{64}{12}$ $\quad A = 32 - \frac{128}{12} = \frac{384 - 128}{12} = \frac{256}{12} = \frac{16}{3}$

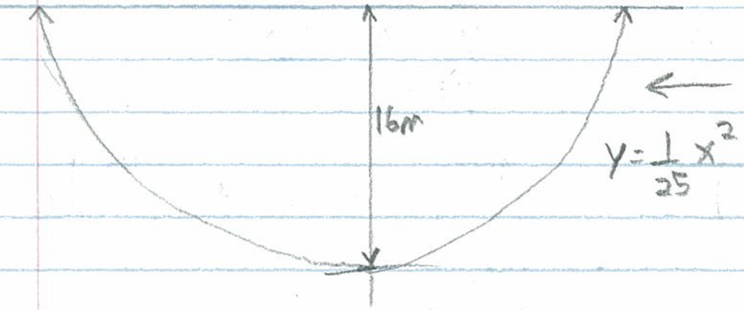
$= \frac{64}{3}$

$\bar{x} = \int_0^4 x(y_2 - y_1)\,dx$ $\quad \bar{x} = \int_0^4 x\left(2x^{1/2} - (-2x^{1/2})\right) dx$

$\bar{x} = \int_0^4 x(4x^{1/2})\,dx = \int_0^4 (4x^{3/2})\,dx = \frac{4x^{3/2 + 2/2}}{5/2} = \frac{8x^{5/2}}{5}\Big|_0^4$

$\frac{8(4)^{5/2}}{5} = \frac{256}{5} \Big/ \frac{64}{3}$ $\Rightarrow \frac{\cancel{256}^{4}}{5} \cdot \frac{3}{\cancel{64}} = \left(\frac{12}{5}, 0\right)$

5) $y = \frac{1}{25}x^2$ $D = 16\,m$ $9.81\ KN/m^3$



$y = \frac{1}{25}x^2$

$25y = x^2$

$x = \sqrt{25y}$

$x = 5y^{1/2}$

$$F = w\int_a^b (\text{depth})(x_2 - x_1)\,dy$$

$$F = 9.81\int_0^{16} (16-y)\left(5y^{1/2} - (-5y^{1/2})\right)dy$$

$$F = 9.81\int_0^{16} (16-y)(10y^{1/2})\,dy = \int_0^{16}\left(160y^{1/2} - 10y^{3/2}\right)dy$$

$$= \frac{160y^{1/2+2/2}}{3/2} - \frac{10y^{3/2+2/2}}{5/2} = \frac{(2)160y^{3/2}}{3} - \frac{(2)10y^{5/2}}{5}$$

$$= \frac{320y^{3/2}}{3} - 4y^{5/2}\Big|_0^{16} = \frac{320(16)^{3/2}}{3} - 4(16)^{5/2} - [0]$$

$$= \frac{20480}{3} - 4096 = \frac{20480 - 12288}{3} = \frac{8192}{3}$$

$$F = 9.81\left(\frac{8192}{3}\right) = 26788 = 26800\ KN$$

6) $y = 0.08x^{3/2}$ $x = 0$ to $x = 100\,m$

$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ $y' = \frac{3}{2}(0.08)x^{3/2 - 2/2}$
$y' = 0.12x$

$S = \int_0^{100} \left(1 + (0.12x^{1/2})^2\right)^{1/2} dx = \int_0^{100} (1 + 0.0144x)^{1/2} dx$

$u = 1 + 0.0144x$ $\frac{1}{0.0144}\int_0^{100} u^{1/2}\, du$
$du = 0.0144\, dx$

$= \frac{u^{1/2 + 2/2}}{3/2} = \frac{2u^{3/2}}{3} = \frac{2(1 + 0.0144x)^{3/2}}{3} \Big|_0^{100}$

$= \frac{1}{0.0144}\left[\frac{2(1 + 0.0144(100))^{3/2}}{3} - \left(\frac{2(1 + 0.0144(0))^{3/2}}{3}\right)\right]$

$= \frac{1}{0.0144}(2.541 - 0.66) = 130.16\, m \cong 130\, m$